Paulton Infant School Calculation Policy

Learning Together, Learning for Life

This policy has been largely adapted from the White Rose Maths Hub Calculation Policy with further material added.

It is a working document and will be revised and amended as necessary.

The National Curriculum 2014

Mathematics is a creative and highly inter-connected discipline that has been developed over centuries, providing the solution to some of history's most intriguing problems. It is essential to everyday life, critical to science, technology and engineering, and necessary for financial literacy and most forms of employment. A high-quality mathematics education therefore provides a foundation for understanding the world, the ability to reason mathematically, an appreciation of the beauty and power of mathematics and a sense of enjoyment and curiosity about the subject.

The aims of this policy

Mastery is for all, and the aim of this policy is to ensure all children leave our school with a secure understanding of the four operations and can confidently use both written and mental calculation strategies in a range of contexts. It aims to ensure consistent strategies, models and images are used across the school to embed and deepen children's learning and understanding of mathematical concepts.

Age Related Expectations

The calculation policy is organised according to Age Related Expectations (ARE) as set out in the National Curriculum 2014. This is a '***mastery curriculum'*** which means that teachers do **not** teach children to the next year group's expectations when they have reached the Age Related Expectations. The children are then challenged to achieve 'mastery' with more sophisticated problems to extend their using and applying skills and to consolidate their knowledge and understanding eg. Using opening up questions.

Providing a context for calculation:

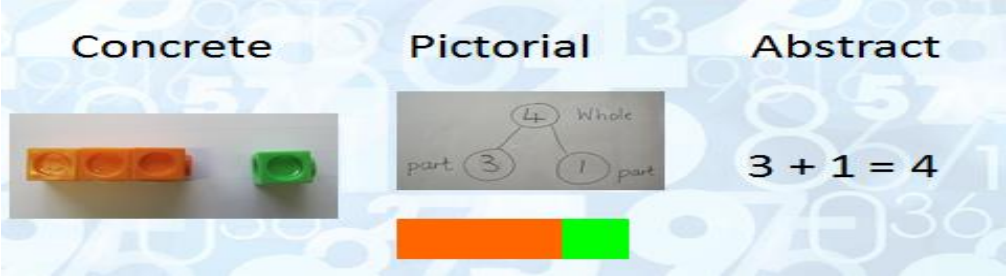
It is important that any type of calculation is given a real life context or problem solving approach to help build children's understanding of the purpose of calculation, and to help them recognise when to use certain operations and methods when faced with problems. This must be a priority within calculation lessons.

How should this policy be used?

This policy has been designed to support the teaching and planning of mathematics in our school. The policy only details the strategies, and teachers must plan opportunities for pupils to apply these; for example, when solving problems, or where opportunities emerge elsewhere in the curriculum. The examples and illustrations are not exhaustive but provide and overall picture of what the mathematics in our school should look like. This is not a scheme of work and must be used in conjunction with our school maths policy and curriculum documents.

Concrete Pictorial Abstract

This policy sets out the progression of strategies and written methods which children will be taught as they develop in their understanding of the four operations. Strategies are set out in a ***Concrete, Pictorial, Abstract (CPA)*** approach to develop children's deep understanding and mastery of mathematical concepts. Children use concrete objects to help them make sense of the concept or problem; this could be anything from real or plastic fruit, to straws, counters or cubes. This is then developed through the use of images, models and children's own pictorial representations before moving on to the abstract mathematics. Children will travel along this continuum again and again, often revisiting previous stages when a concept is extended. It is also worth noting that if a child has moved on from the concrete to the pictorial, it does not mean that the concrete cannot be used alongside the pictorial. Or if a child is working in the abstract, 'proving' something or 'working out' could involve use of the concrete or pictorial.



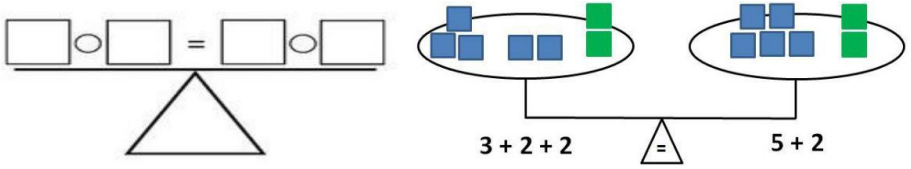
Similarly, although the strategies are taught in a progressive sequence, they are designed to equip children with a 'tool box' of skills and strategies that they can apply to solve problems in a range of contexts. So as a new strategy is taught it does not necessarily supersede the previous ones, but builds on prior learning to enable children to have a variety of tools to select from. As children become increasingly independent, they will be able to and must be encouraged to select those strategies which are most efficient for the task.

The strategies are separated into the 4 operations for ease of reference. However, it is intended that addition and subtraction, and multiplication and division will be taught together to ensure that children are making connections and seeing relationships in their mathematics. Therefore, some strategies will be taught simultaneously, for example, counting on (addition) and counting back (subtraction).

Children should be moved through the strategies at a pace appropriate to their age related expectations as defined in the EYFS and NC. Effective teaching of the strategies relies upon increasing levels of number sense, fluency and ability to reason mathematically. Children must be supported to gain depth of understanding within the strategy through the CPA approach and not learn strategies as a procedure.

Teaching equality

It is important that when teaching the 4 operations that equality (=) is also taught appropriately. Misconceptions that = means that children must 'do something' and that it indicates that an answer is needed are common and must be addressed early on. Teachers should present children with number sentences and problems which place the = sign in different positions, different context and include missing box problems. For example, ?+4=7; 7=3+?; <, >, or = 5+6__ 7+4. In the concrete phase. scales and Numicon provide a useful resource to demonstrate equality. Pictorial representations of equality can be used as shown below:



Importance of vocabulary

The 2014 National Curriculum places great emphasis on the importance of pupils using the correct mathematical language as a central part of their learning. Children will be unable to articulate their mathematical reasoning if they lack the mathematical vocabulary required to do so. The use of whole sentences know as **stem sentences** are crucial – these are identified in the policy in green text or box outlines. It is therefore essential that teaching using the strategies outlined in this policy is accompanied by the use of appropriate mathematical vocabulary. New vocabulary should be introduced in a suitable context (for example, with relevant real objects, apparatus, pictures or diagrams) and explained carefully. High expectations of the mathematical language used are essential, with teachers modelling and only accepting what is correct. For example:

Previous Curriculum (NC 2000)	Current curriculum (NC 2014)
Units	Ones (1s)
Is equal to	equals
O – letter o	zero
Sum(s)	Number sentence (Calculation sentence)

End of year expectations for calculations

EYFS	YEAR 1	YEAR 2
Count reliably with numbers from one to 20. Place numbers in order. Say which number is one more or one less than a given number. Using quantities and objects, add two single-digit numbersand count on to find the answer. Using quantities and objects, subtract two single-digit numbers and back to find the answer. Solve problems, including doubling, halving and sharing.	Read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs Represent and use number bonds and related subtraction facts within 20 Add and subtract one-digit and two-digit numbers to 20, including zero Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = ? – 9. Solve one-step problems involving multiplication and division, by calculating the answer using concrete objects Solve one-step problems involving multiplication and division using pictorial representations and arrays with the support of the teacher	Solve problems with addition and subtraction: using concrete objects and pictorial representations, including those involving numbers, quantities and measures Apply increasing knowledge of mental and written methods Recall and use addition and subtraction facts to 20 fluently Derive and use related facts up to 100 Add and subtract numbers using concrete objects, pictorial representations, and mentally, including: - a two-digit number and ones - a two-digit number and tens - two two-digit numbers - adding three one-digit numbers Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication (×), division (÷) and equals (=) signs Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts

Progression in Calculations - Addition

EYFS - Reception

Before addition can be introduced, children in Reception build on concepts taught in Nursery by working through the number objectives in the 40 – 60 month band of Development Matters. Children need to have a secure knowledge of number in order to begin addition. Children are then introduced to the concept of addition through practical games and activities. Children act out addition sums to physically add two groups of objects together and use arm gestures to represent the signs + and =.

This is reinforced by opportunities provided in the outdoor area for the children to use addition e.g. adding together groups of building blocks, twigs etc. Children build on their previous knowledge of 'more' by learning that adding two groups of objects together gives them a larger number (more objects). Adults model addition vocabulary supported by age appropriate definition. An example of this is "addition means we add two groups together / we put 2 lots of objects together. Equals means we find out how many we have got altogether. 3 add 2 equals 5! We have got 5 altogether". Adults support children in recording their addition sums in the written form on whiteboards and in their maths books.

Reception Addition

Early Years Add with numbers up to 10

Count reliably up to 10 every day objects using 1:1 correspondence

How many Teddies?

There are 2 teddies and 3 teddies. That's 5 teddies altogether. Watch me count them!

Begin adding by combining two sets of objects into one group (5 cubes and 3 cubes). Add 2 groups together by counting all of them. Understand that addition means putting groups together.

2 + **3** = **5**

Only when ready use numbered number lines to add, by counting on in ones. Children are encouraged to use methods in the inside and outside environment. They develop ways of recording calculations using pictures and objects. They will add two single digit numbers using objects and by counting on, on a number line.

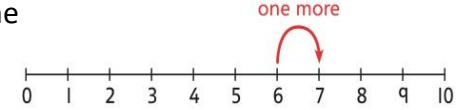
To count one more, 

then several more, on a number line

For example: 6 + 1 = 7

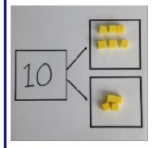
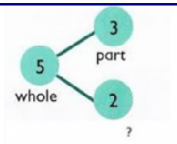
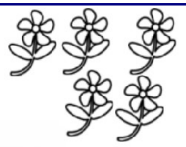
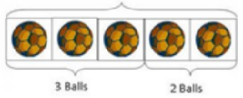
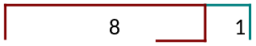
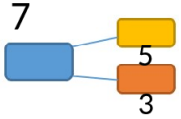
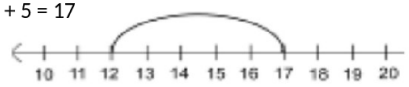
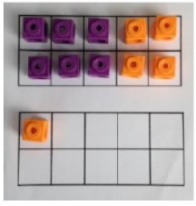
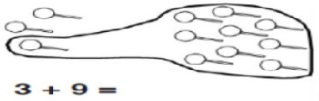
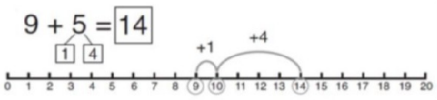
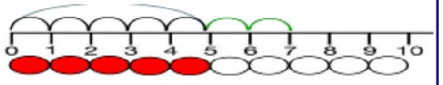
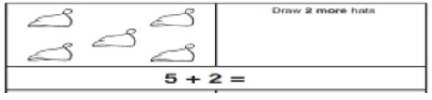
One more than 6 is 7.

7 is one more than 6.

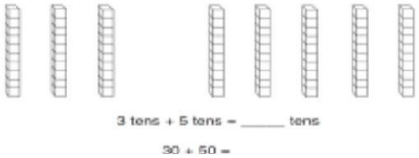
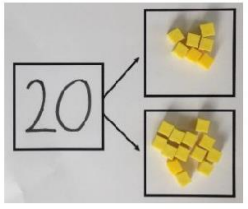
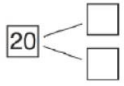
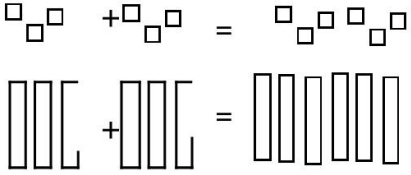
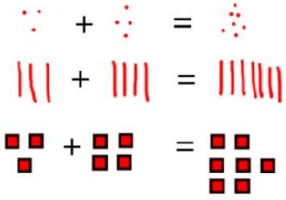
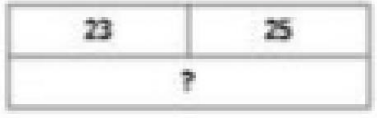


Children are encouraged to make use of fingers as these are a constantly available resource for calculations at this level.

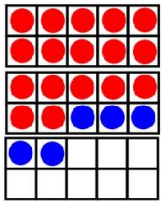
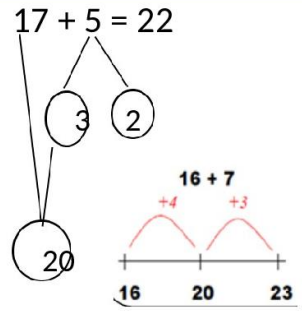
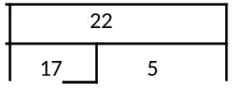
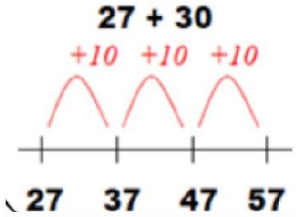
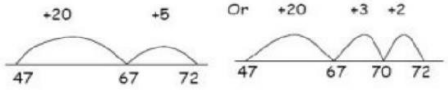
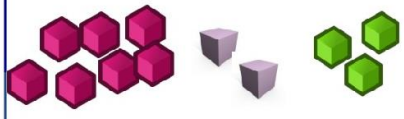
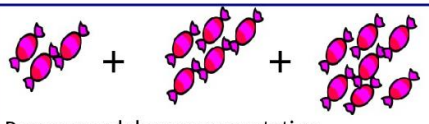
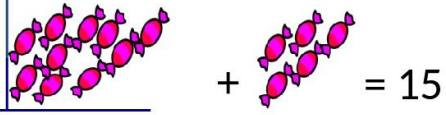
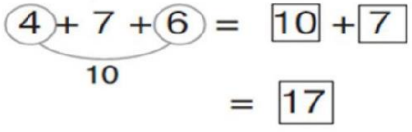
Y1 Addition

Objective & Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part- whole model *The whole is 7.* *The parts are 4 and 3.*	   Use part part whole model. Use cubes to add two numbers together as a group or in a bar. 	   Use pictures to add two numbers together as a group or in a bar. 	4 + 3 = 7  10= 6 + 4 Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on 	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17  Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10. *This is an essential skill for column addition later.* *6 + 5* *6 and 4 make 10. So 6 add 5 is 10 and one more*	 6 + 5 = 11  Start with the bigger number and use the smaller number to make 10. Use ten frames.	 Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10. 	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?
Represent & use number bonds and related subtraction facts within 20	 2 more than 5.	 	Emphasis should be on the language *'1 more than 5 is equal to 6.'* *'2 more than 5 is 7.'* *'8 is 3 more than 5.'*

Y2 Addition

Objective & Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten *3 tens and 2 tens makes 5 tens which is 50.*	50 = 30 + 20  Model using dienes and bead strings	 Use representations for base ten.	20 + 30 = 50 70 = 50 + 20 40 + □ = 60
Use known number facts Part part whole	 Children explore ways of making numbers within 20	 □ + □ = 20 20 − □ = □ □ + □ = 20 20 − □ = □	□ + 1 = 16 16 − 1 = □ 1 + □ = 16 16 − □ = 1
Using known facts *I know that 3 add 3 equals 6 so 3 tens add 3 tens equals 6 tens which is 60*		 Children draw representations of H,T and O	3 + 4 = 7 *leads to* 30 + 40 = 70 *leads to* *I know that 4 + 3 = 7. So, I know that 4 tens add 3 tens is 7 tens.*
Bar model	 3 + 4 = 7	 7 + 3 = 10	 23 + 25 = 48

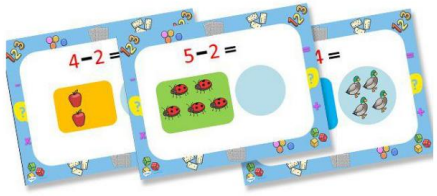
Y2 Addition

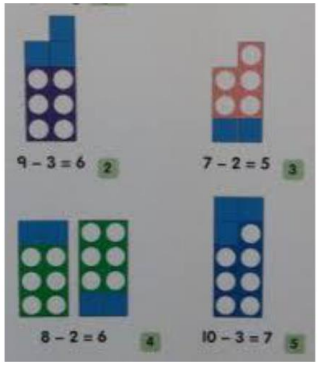
Objective & Strategy	Concrete	Pictorial	Abstract
Add a two digit number and ones *I know I can partition 5 into 3 and 2. SO 17 add 3 makes 20 and 20 add 2 makes 22*	 17 + 5 = 22 Use ten frame to make 'magic ten Children explore the pattern. 17 + 5 = 22 27 + 5 = 32	Use part part whole and number line to model. 	17 + 5 = 22 Explore related facts 17 + 5 = 22 5 + 17 = 22 22—17 = 5 22—5 = 17 
Add a 2 digit number and tens *25 add 10 means the tens number will become 30 and the ones will be the same.*	 25 + 10 = 35 Explore that the ones digit does not change		27 + 10 = 37 27 + 20 = 47 27 + □ = 57
Add two 2-digit numbers *5 ones add 7 ones equals 12 ones.* *2 tens add 4 tens makes 6 tens which is 60. The total is 72*	 Model using dienes , place value counters and numicon	 Use number line and bridge ten using part whole if necessary.	25 + 47 20 + 5 40 + 7 20 + 40 = 60 5+ 7 =12 60 + 12 = 72
Add three 1-digit numbers *I can see 7 pink cubes added to the 3 green cubes will make 10. Then add the 2 grey cubes to 10 will make 12.*	 Combine to make 10 first if possible, or bridge 1o then add third digit	 Regroup and draw representation. 	 Combine the two numbers that make/ bridge ten then add on the third.

Progression in Calculations- Subtraction

EYFS - Reception

Before subtraction can be introduced, children in Reception build on concepts taught in Pre – school settings by working through the number objectives in the 40 – 60 month band of Development Matters. Children need to have a secure knowledge of number in order to begin subtraction. Children are then introduced to the concept of subtraction through practical games and activities. Children act out subtractions to physically subtract a number of objects from a group. Children use arm gestures to represent the signs - and =. This is reinforced by opportunities provided in the outdoor area for the children to count e.g. counting building blocks, twigs etc. Children build on their previous knowledge of 'less' by learning that subtracting means taking away a certain number of objects from a group (leaving them with less objects). Adults model subtraction vocabulary supported by age appropriate definition. An example of this is "subtraction means we take away objects from a group / we have 11 got less objects now. Equals means we find out how many we have got left. Wow! We have only got 3 left!" Adults support children in recording their subtractions in the written form on whiteboards and in their maths books.



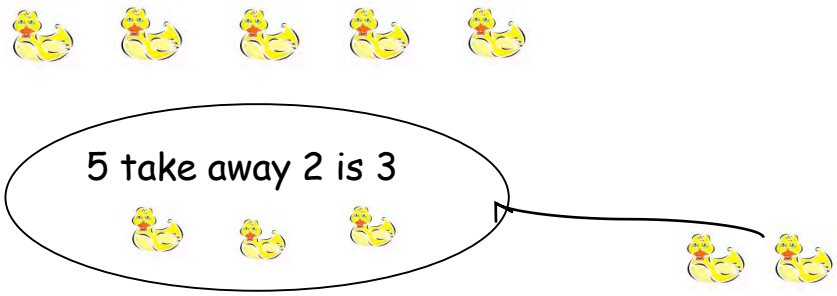


Reception Subtraction

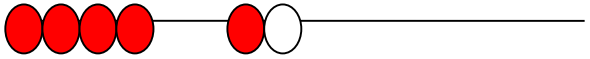
Early Years Subtract from numbers up to 20

In the early stages, children will be taught to 'take away' one or two objects and find the new total.

For example: 5—3 = 2



Bead strings or bead bars can be used to illustrate subtraction by counting back a single digit.



Find a numerical difference by comparing 2 sets e.g. sets of cubes.

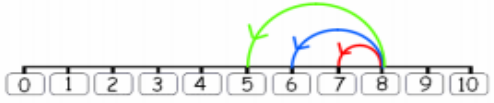
The difference between 8 and 5 is 3

The next step for children is to be able to work out one less or several less on a number line.

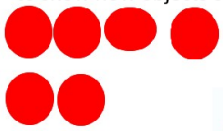
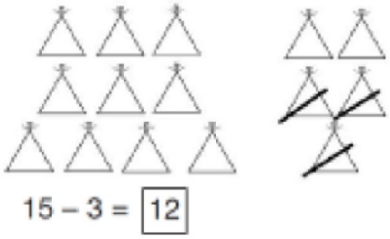
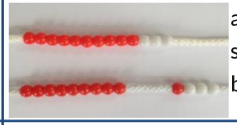
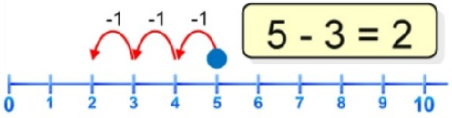
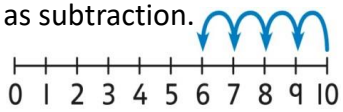
1 less than 8 is 7 — 8 in my head, count back 1 is 7

2 less than 8 is 6 — 8 (7, 6)

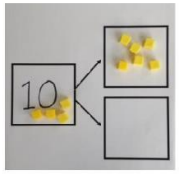
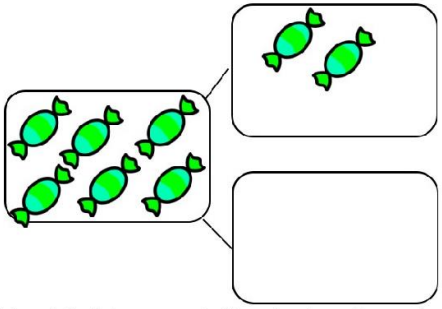
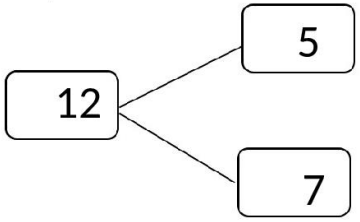
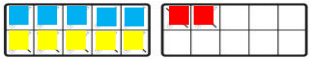
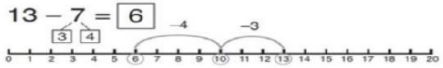
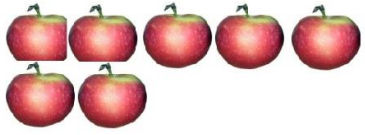
3 less than 8 is 5 — 8 (7, 6, 5)



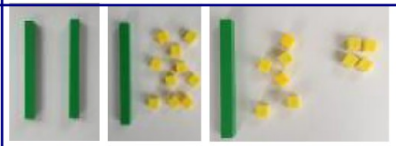
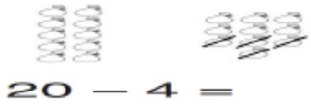
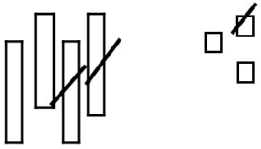
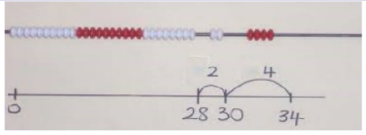
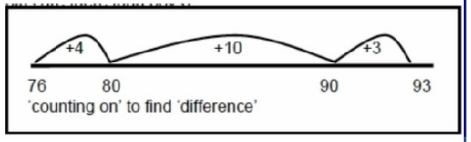
Y1 SUBTRACTION.

Objective & Strategy	Concrete	Pictorial	Abstract
Taking away ones. *2 less than 6 is 4.* *6 subtract 2 is 4.*	Use physical objects, counters , cubes etc to show how objects can be taken away.  6—4 = 2  4—2 = 2	 Cross out drawn objects to show what has been taken away.	7—4 = 3 16—9 = 7
Counting back	 Move objects away from the group, counting backwards.  Move the beads along the bead string as you count backwards.	 Count back in ones using a number line.	Put 13 in your head, count back 4. What number are you at?
Find the Difference	**Finding the difference** Arrange two groups so that the difference between the groups can be worked out.  *8 is 2 more than 6.* *6 is 2 less than .* *The difference between 8 and 6 is 2.*	**Finding the difference** Represent objects using sketches or counters to support finding the difference.  *5 – 4 = 1 The difference between 5 and 4 is 1.*	**Finding the difference** Children understand 'find the difference' as subtraction.  *10 – 4 = 6* *The difference between 10 and 6 is 4.*

Y1 SUBTRACTION.

Objective & Strategy	Concrete	Pictorial	Abstract
Represent and use number bonds and related subtraction facts within 20 Part Part Whole model	 Link to addition. Use PPW model to model the inverse. If 10 is the whole and 6 is one of the arts, what s the other part? 10—6 = 4	 Use pictorial representations to show the part.	Move to using numbers within the part whole model. 
Make 10	For example: 12 – 7 Arrange objects into a 10 and some 1s, then decide on how to split the 7 into parts  *7 is 2 and 5, so I take away the 2 and then the 5. this leaves me with 5*	13—7  Jump back 3 first, then another 4. Use ten as the stopping point.	16—8 How many do we take off first to get to 10? How many left to take off? *I can take away 6 to make 10 and then take away the remaining 2 from 10 this will give an answer of 8*
Bar model *5 take away 2 leaves another part which would be 3*	 5—2 = 3		 10 = 8 + 2 10 = 2 + 8 10—2 = 8 10—8 = 2

Y2 SUBTRACTION.

Objective & Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	 Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'		20—4 = 16
Partitioning to subtract without re-grouping. *'Friendly numbers'*	34—13 = 21   Use Dienes to show how to partition the number when subtracting without regrouping. 	Children draw representations of Dienes and cross off.  43—21 = 22	43—21 = 22
Make ten strategy *Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.*	 34—28 Use a bead bar or bead strings to model counting to next ten and the rest.	 Use a number line to count on to next ten and then the rest.	93—76 = 17

Progression in Calculations - Multiplication

EYFS - Reception

By the end of Reception, children are expected to understand the concept of doubling and to be able to double a number up to

Before doubling can be introduced, children need to have a secure knowledge of counting, number facts and addition in order to double. Children are then introduced to the concept of doubling through practical games and activities, including the use of the outdoor areas. Children act out 'doubling' by physically add two equal groups together to find out the 'doubles' answer.

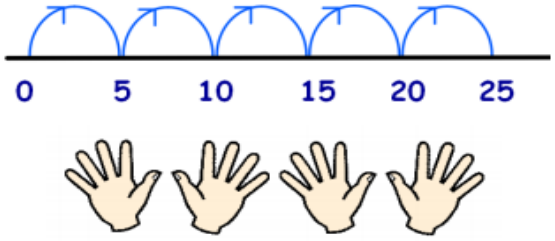
Reception
Multiplication

Early Years Multiply with concrete objects using repeated addition.

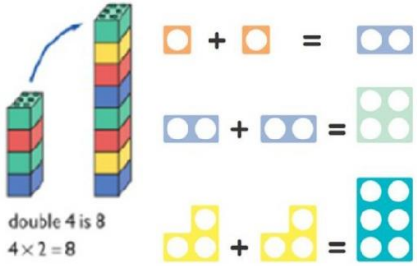
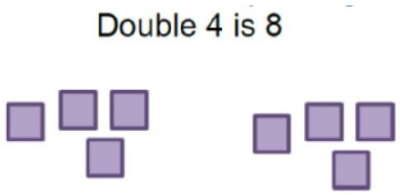
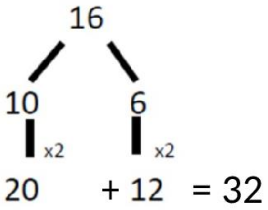
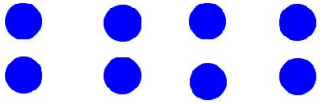
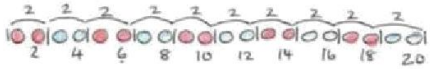
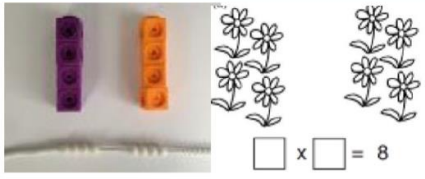
By the end of Reception, children are expected to understand the concept of doubling and to be able to double a number up to 10. Before doubling can be introduced, children need to have a secure knowledge of counting, number facts and addition in order to double.

Real life contexts and use of practical equipment to count in repeated groups of the same size; counting in twos; fives; tens. Also chanting in 2s, 5s and 10s.
They are encouraged to solve real life problems e.g. If one pair of wellington boots = 2 then 3 pairs = 6
They are encouraged to draw pictures and represent their mathematical thinking through various representations e.g. Bead strings, numicon, cubes.

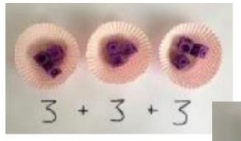
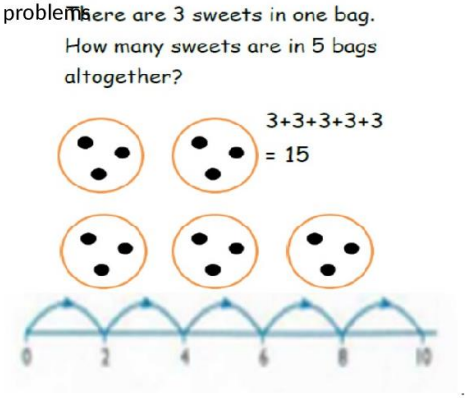
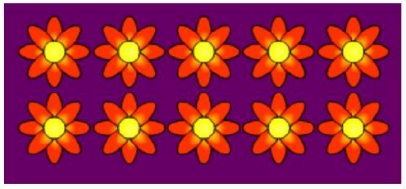
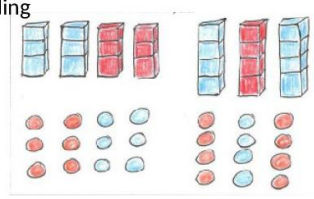
They will work on practical problem solving activities involving equal sets or groups including the introduction to finding doubles.



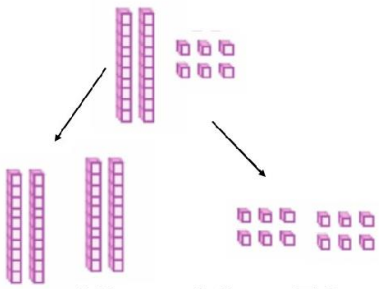
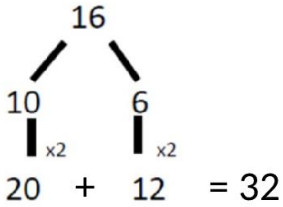
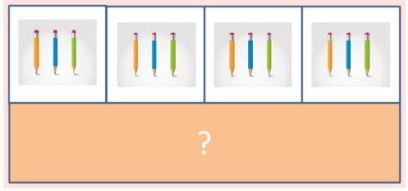
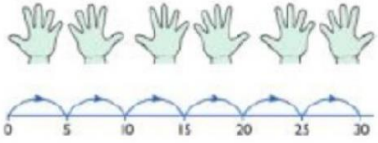
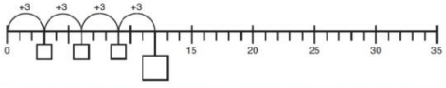
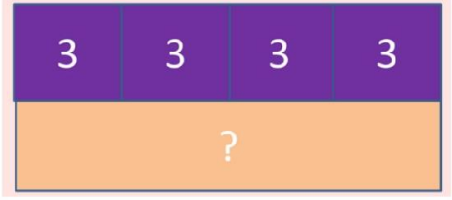
Y1 MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Use practical activities using manipultives including cubes and Numicon to demonstrate doubling  	Draw pictures to show how to double numbers  	Partition a number and then double each part before recombining it back together. 
Counting in multiples *2, 4, 6, 8*	Count the groups as children are skip counting, children may use their fingers as they are skip counting.  	 Children make representations to show counting in multiples.  	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30
Making equal groups and counting the total *2 equal groups of 4 makes a total of 8*	 ☐ x ☐ = 8 Use manipulatives to create equal groups.	Draw  to show 2 x 3 = 6 Draw and make representations	2 x 4 = 8 *2 groups of 4 makes 8.* *2 times 4 equals 8.* *4 multiplied by 2 equals 8*

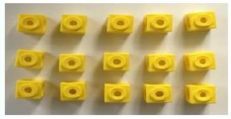
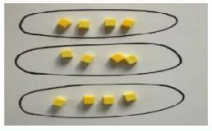
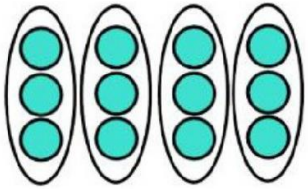
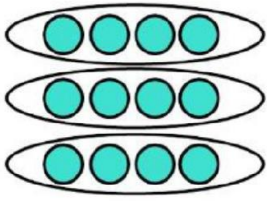
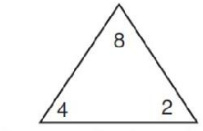
Y1 MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract
Repeated addition	   Use different objects to add equal groups	Use pictorial including number lines to solve problems. 	Write addition sentences to describe objects and pictures. 
Understanding arrays	Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc. 	Draw representations of arrays to show understanding 	3 x 2 = 6 2 x 5 = 10
	I can see 2 groups of 5 *and I can see 5 groups of 2*		*3 groups of 2 is 6* *3, 2 times is 6* *2 groups of 5 is 10* *2, 5 times equals 10*

Y2 MULTIPLICATION

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	Model doubling using dienes and PV counters.  40 + 12 = 52	Draw pictures and representations to show how to double numbers	Partition a number and then double each part before recombining it back together. 
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.  5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40  	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.   	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25 , 30 $4 \times 3 = \square$

Y2 MULTIPLICATION X

Objective & Strategy	Concrete	Pictorial	Abstract
Multiplication is commutative *I can see four groups of three.* *I can see three groups of four*	Create arrays using counters and cubes and Numicon.    Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.  	Use representations of arrays to show different calculations and explore commutativity.  	12 = 3 × 4 12 = 4 × 3 Use an array to write multiplication sentences and reinforce repeated addition.  5 + 5 + 5 = 15 3 + 3 + 3 + 3 + 3 = 15 5 x 3 = 15 3 x 5 = 15
Using the Inverse *This should be taught alongside division, so pupils learn how they work alongside each other.*		 □ × □ = □ □ × □ = □ □ ÷ □ = □ □ ÷ □ = □	2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8÷ 2 Show all 8 related fact family sentences.

Progression in Calculations - **Division**

EYFS - Reception

By the end of Reception, children are expected to understand the concept of halving and sharing. Before this can be introduced, children need to have a secure knowledge of counting backwards, number facts and subtraction in order to halve and share. Children are then introduced to the concept of halving and sharing through practical games and activities. They act out 'halving and sharing' through activities such as

sharing food for their Teddy Bear's Picnic, sharing resources equally to play a game. This is reinforced by opportunities provided in the outdoor area for the children to halve and share out objects such as building blocks, twigs etc.

Reception Division

Early Years Group and share small quantities

Children will understand equal groups and share items out in play and problem solving. They will count in 2s and 10s and later in 5s. Halve objects and numbers by sharing.

Using objects, diagrams and pictorial representations to solve problems involving **both grouping and sharing**.

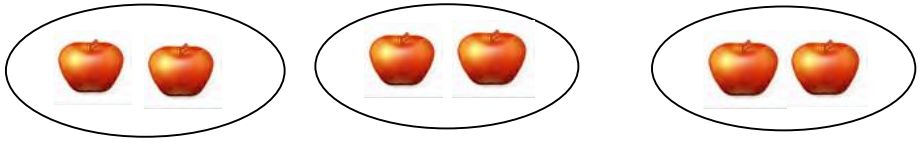
Find **half** of a group of objects by sharing into 2 equal groups.

Sharing into equal groups - Share objects into equal groups and count how many in each group.

Grouping - Children will understand equal groups and share items out in play and problem solving. They will count in 2s and 10s and later in 5s.

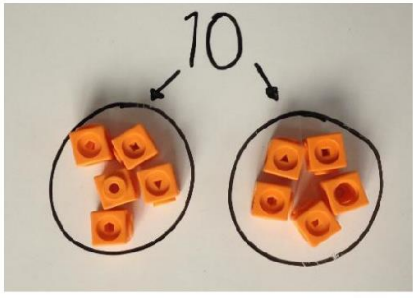
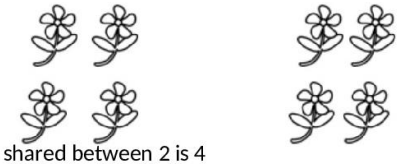
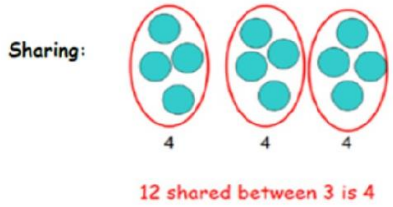
How many groups of 4 can be made with 12 faces ? = **3**

4 groups of 3 can be made with 12 faces

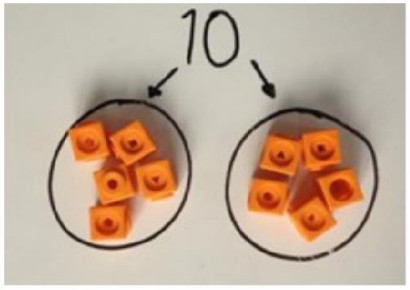
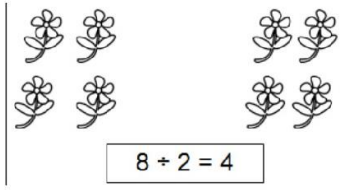
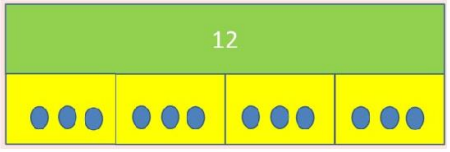
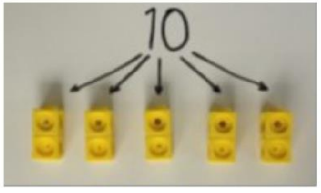
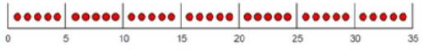
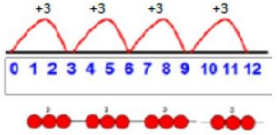
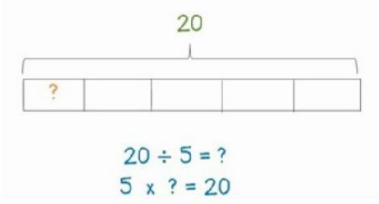


3 groups of 2 can be made with 6 apples.
I can share 6 apples into 3 groups there will be 2 in each group.

Y1 ÷ DIVISION

Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing *Use Gordon ITPs for modelling*	   I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities.  8 shared between 2 is 4  	12 shared between 3 is 4 *12 divided between 3 equals 4* *When I share 12 between 3 I have 4 in each group*

Y2 DIVISION ÷

Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing	 I have 10 cubes, can you share them equally in 2 groups? *When I share 10 cubes into 2 groups there will be 5 in each group*	Children use pictures or shapes to share quantities.  Children use bar modelling to show and support understanding.  12 ÷ 4 = 3	12 ÷ 3 = 4 *12 divided between 3 equals 4* *When I share 12 between 3 I have 4 in each group*
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  	Use number lines for grouping  12 ÷ 3 = 4 Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group. 	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group? *28 divided into groups of 7 gives me 4 groups* *There are 4 groups of 7 in 28*